Finite Temperature Lattice QCD with Clover Fermions *

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We report on our simulation of finite temperature lattice QCD with two flavors of $\mathcal{O}(a)$ Symanzik-improved fermions and $\mathcal{O}(a^2)$ Symanzik-improved glue. Our thermodynamic simulations were performed on an $8^3 \times 4$ lattice, and we have performed complementary zero temperature simulations on an $8^3 \times 16$ lattice. We compare our results to those from simulations with two flavors of Wilson fermions and discuss the improvement resulting from use of the improved action.

1. INTRODUCTION

The study of finite temperature QCD with Wilson-type quarks is desirable in order to estimate any systematic errors of similar simulations with Kogut-Susskind quarks. However, Wilson thermodynamics has proved to be difficult and burdened with lattice artifacts [1]. It is plausible that an action which converges to the continuum action faster in the $a \to 0$ limit would be cured of such spurious effects.

2. ACTION

For the gauge action, we start with the one loop, on-shell Symanzik improved action derived by Lüscher and Weisz [2]. We implement the tadpole improvement scheme in order that lattice perturbation theory be more convergent [3,4]. We choose to define the “mean link” u_0 and the strong coupling constant α_s through the plaquette [3–5]:

The coefficients of the rectangle operator and the twisted 6-link operator, $\beta_{\rm rect}$ and $\beta_{\rm twist}$ respectively, are given in terms of the coefficient of the plaquette β and u_0 as in [4]:

$$\beta_{\rm rect} = -\frac{\beta}{20u_0^2}\Big(1 - 0.6264\ \ln(u_0)\Big) \qquad (1)$$

$$\beta_{\rm twist} = \frac{\beta}{u_0^2}\ 0.04335\ \ln(u_0). \qquad (2)$$

In practice, we estimate u_0 in a self-consistent manner: we tune it so that it agrees with the fourth root of the space-like plaquettes.

The Wilson fermion action has errors of $\mathcal{O}(a)$. The Symanzik improvement program is used to improve the action [6]. After tadpole improvement the fermion action is

$$S_f = S_W - \frac{\kappa}{u_0^3}\sum_x\sum_{\mu<\nu}\Big[\overline{\psi}(x)\ i\sigma_{\mu\nu}F_{\mu\nu}\psi(x)\Big], \qquad (3)$$

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where S_W is the usual Wilson fermion action, and $iF_{\mu\nu}$ is the familiar clover-shaped link operator.

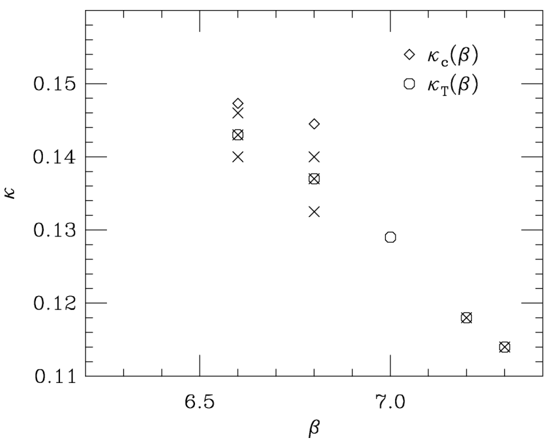

Figure 1. Phase diagram of Symanzik-improved action. Octagons represent the $N_t = 4$ thermal crossover, and diamonds indicate estimates of vanishing pion mass. Zero temperature simulations were performed at the crosses.

3. RESULTS

Our thermodynamics simulations were done on an $8^3 \times 4$ lattice at six fixed values of β while varying κ across the thermal crossover (tuning u_0 self-consistently at each parameter set). We used the hybrid Monte Carlo algorithm and collected data from at least 1000 trajectories for the simulations in the crossover region. Furthermore, zero temperature simulations on an $8^3 \times 16$ lattice were performed in order to provide hadron masses in the region of the thermal crossover line. The phase diagram (figure 1) summarizes our run parameters.

Figure 2 shows the Polyakov loop as a function of κ for the six values of β. One can observe that the transition appears steeper for stronger coupling: a feature also present in $N_t = 4$ Wilson thermodynamics [7]. Still, the crossover for the improved action does not appear to be as steep as for the unimproved action.

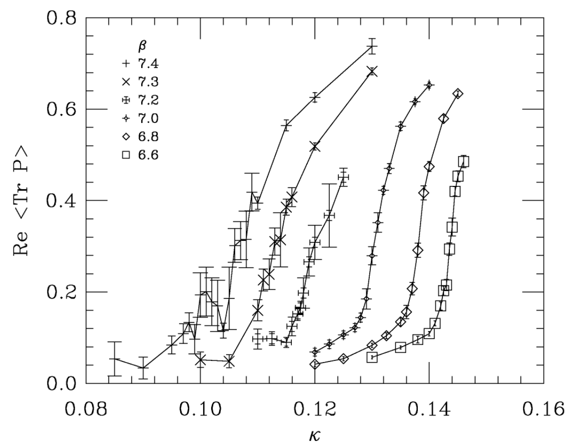

Figure 2. Polyakov loop vs. hopping parameter for $8^3 \times 4$ improved Wilson thermodynamics.

One would like to make direct comparison between the two actions of their respective crossover behavior without depending on the bare parameters. In this work, we use measurements of the lattice pion mass squared at values of κ near the crossover. Then, we can plausibly overlay curves of thermodynamic observables for two actions run at comparable m_π/m_ρ. Below we list m_π/m_ρ along the $N_t = 4$ crossover for both clover and Wilson [8] actions.

Clover			Wilson		
β_{Cl}	κ_{Cl}	m_π/m_ρ	m_π/m_ρ	β_{W}	κ_{W}
6.6	0.143	0.725(24)	0.708(7)	4.76	0.19
6.8	0.137	0.831(10)	0.836(5)	4.94	0.18
7.2	0.118	0.968(4)	0.899(4)	5.12	0.17
7.3	0.114	0.970(3)	0.943(5)	5.28	0.16

Using measurements of the pion mass near the crossover region [9], we can interpolate in order to estimate $(am_\pi)^2$ as a function of $1/\kappa$. Then, we can plot the thermodynamic observables against the pion mass squared. This shows that the crossover is indeed smoother for $N_t = 4$ clover than $N_t = 4$ Wilson (see figure 3).

Finally, the confinement-deconfinement temperatures for different two-flavor lattice actions are shown in figure 4. One consequence of our improvement scheme is to lower the Wilson $N_t = 4$

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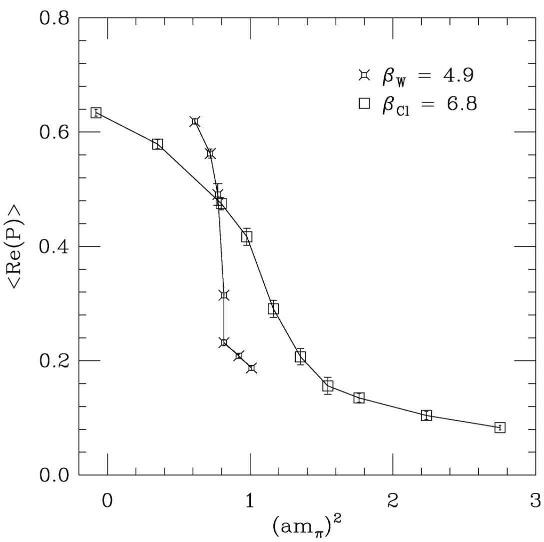

Figure 3. Polyakov loop vs. pion mass squared. The crossovers ($N_t = 4$) for both actions, at the couplings shown, occur at the same $\pi - \rho$ mass ratio: $m_\pi/m_\rho = 0.83$.

critical temperature at a given mass ratio. This brings the calculation of T_c/m_ρ into better agreement with $N_t = 6$ Wilson and with staggered fermion thermodynamics. There is one important caveat: we do not yet know if the clover simulations have reached a plateau in m_π/m_ρ. If T_c/m_ρ continues to rise at lower m_π/m_ρ (lower β) then the aforementioned agreement is accidental. Furthermore, we should remember that T_c/m_ρ tends toward zero as m_π/m_ρ approaches unity, *i.e.* as $m_q \to \infty$. Measurements of the string tension will provide a scale which is insensitive to the quark mass.

4. CONCLUSIONS

We have shown that the $N_t = 4$ thermal crossover is smoother for the Symanzik-improved action. It could be that T_c/m_ρ is in better agreement with staggered fermion results; however running at the thermal crossover at lower m_π/m_ρ is needed to confirm this.

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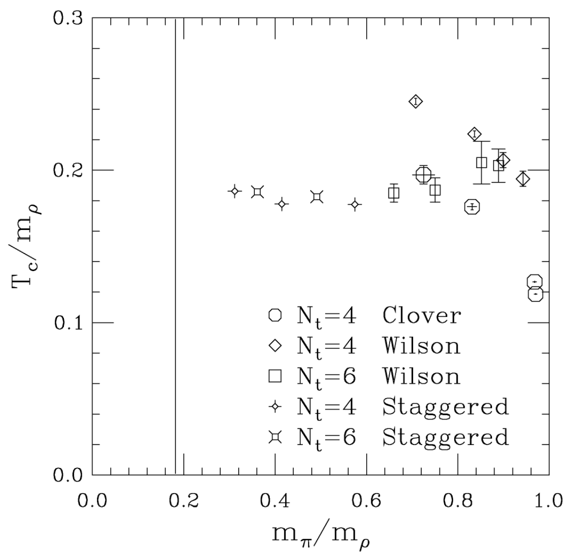

Figure 4. Crossover temperature in units of the ρ mass vs. m_π/m_ρ.

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